Convergence

Let $W_1, W_2, \ldots$ and W be real-valued random variables.

- Say the W_n *converge in distribution* to W with distribution function F (and write $W_n \xrightarrow{d} W$) if, for all t at which F is continuous,

$$\mathbb{P}(W_n \leq t) \to F(t).$$

- We say the W_n *converge in probability* to W and write $W_n \xrightarrow{p} W$ if for all $\epsilon > 0$,

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$$W_n \xrightarrow{p} W \Rightarrow W_n \xrightarrow{d} W$$

$$W_n \xrightarrow{d} c \text{ with } c \text{ deterministic } \Rightarrow W_n \xrightarrow{p} W.$$

Basic results

Theorem (Law of large numbers (LLN))

If $W_1, W_2, \ldots$ *are i.i.d. real-valued random variables and* $\mathbb{E}(W_1) = \mu < \infty$*, then as* $n \to \infty$*,*

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Suppose the sequence of random variables $(W_n)_{n=1}^{\infty}$ *is such that* $W_n \xrightarrow{p} W$. *Let* $f : \mathbb{R} \to \mathbb{R}$ *be continuous at every point in a set* C *with* $\mathbb{P}(W \in C) = 1$. *Then* $f(W_n) \xrightarrow{p} f(W)$.

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Lemma (Slutsky's lemma)

Let $(U_n)_{n=1}^{\infty}$ *and* $(W_n)_{n=1}^{\infty}$ *be sequences of random variables where* $U_n \xrightarrow{d} U$ *and* $W_n \xrightarrow{p} c$ *for random variable* $U \in \mathbb{R}$ *and deterministic* $c \in \mathbb{R}$. *Then*

1. $U_n + W_n \xrightarrow{d} U + c$,
2. $U_n W_n \xrightarrow{d} Uc$,
3. $U_n / W_n \xrightarrow{d} U/c$ *if* $c > 0$.